

Exploring Terrain of the Mind

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By Rebecca Bynum (June 2006)

Book Review

[Unknown Quantity \(A Real and Imaginary History of Algebra\)](#)

By John Derbyshire

John Henry Press, 374 pp.

Have you ever taken a really great course in college that altered your view of reality forever? Did you ever have a teacher who could bring clarity to a complex subject and whet your thirst for more? Have you ever had a “wow moment” so staggering it sent energy rushing through your entire body and you could barely breathe? If you’ve had experiences like that before, you will know what’s in store here, if not, fasten your seatbelts and prepare for a journey of discovery that will take you into the farthest reaches of modern mathematical thought under the guidance of a wise and thoughtful teacher.

Mr. Derbyshire one of those very rare writers who seems to be continually conscious of how his words will be received by the reader. He is respectful without being presumptuous and so provides the necessary background for people like myself who have very little mathematical experience. I found I could easily follow the main action in the book, which is rightly centered on abstract mathematics, but that the story did not require my complete comprehension of all the details in every equation: a big plus for anyone who might normally be intimidated by a book like this.

Derbyshire thankfully provides amazingly clear primers for the semi-mathematically-literate, like myself, (why didn’t anybody explain it like this before?) and the overall ordering of the book makes for easy reference which the author uses to good effect, cutting down to zero unnecessary redundancy. He also does an excellent job of pacing and seems to know when the mind of the reader will naturally tire from the mental effort math requires, interspersing beautiful miniatures and telling vignettes of the very human and often entertaining lives

of those involved with extending the boundaries of algebra throughout the ages. This makes for a nicely paced read alternating between periods of intense concentration and light entertainment for a very satisfying overall effect.

The story of algebra is actually the story of the gradual freeing of logic from words and resulting in the exploration of what logic does when so freed. Algebraic objects, structures and processes exist in a purely mental realm and exist there whether we are aware of them or not. They are perhaps the nearest thing human beings can know as examples of "eternal truth," for they seem to be completely independent of both time and space. Yet even the most abstract concepts reached purely algebraically have later been shown to describe many actual processes in time and space, lending credibility to the notion of mind as reality in itself and that mathematics lends us the ability to describe mind as well as matter.

Inversely, algebra also illustrates what seem to be a requirement of the human mind for some sort of framework within the boundaries of which logical thought can function, and thus much of this story centers on the difficulty historically of breaching those parameters and extending them. For example, writes Derbyshire:

The peculiarity of Greek mathematics is that prior to Diophantus [3rd century father of modern algebra] it was mainly geometrical. The usual reason given for this, which sounds plausible to me, is that the school of Pythagoras (late 6th Century BCE) had the idea to found all mathematics – and music and astronomy – on number but that the discovery of irrational numbers so disturbed the Pythagoreans, they turned away from arithmetic, which seemed to contain numbers that could not be written to geometry, where such numbers could be represented infallibly by the lengths of line segments.

As a result of this apparent human tendency to cling to the observable material realm, the development of mathematics was absolutely glacial, noting very little progress over vast swaths of time along with the occasional retrogression. Al-Kwarizmi, whose name is thrown around rather freely these days as the "inventor of algebra" for example, was actually doing work far inferior to the mathematics done by Diophantus 600 years before. In fact, this book should lay to rest the

entire idea that mathematics is invented at all. The symbols used in math are arbitrarily invented to be sure, but the logic they help to uncover is anything but.

Mr. Derbyshire gives greater consideration to another medieval Muslim scholar, Omar Khayyam, who mounted a serious assault on the cubic equation, though even at this late date (11th century CE) the elegant algebraic notation of Diophantus including the concept of negative numbers were still laying around unused, so Khayyam worked in cumbersome word problems the same way the ancient Babylonians had.

Our modern numbering system, including the crucial zero component, comes down to us from Hindu India though the work of a thirteenth century Italian living in what was then Muslim controlled territory. Leonardo of Pisa opened his book *Liber abbaci* with the following words:

“There are nine figures from the Indians: 9 8 7 6 5 4 3 2 1. With these nine figures, and with the sign 0 which in Arabic is called *zephirum*, an number can be written, as will be demonstrated.”

Look how late this development is! Serious mathematics had been pursued from the time of Abrahamic Mesopotamia and ancient Egypt in the 19th century BC and here we are in the 13th century AD finally grasping a coherent numeric notation, something we take absolutely for granted today. How fragile these things are! How precious and how easily lost!

It seems numbers themselves had to become “real” to us as written entities before we could accept their purely imaginary or mindal aspects. Each push into the unknown required the courage of individuals to *imagine* the greater set, or broader organizing principle, growing step by step from the natural to the complex number systems, with each concept encompassing and expanding the framework preceding it. Derbyshire, as always, supplies the perfect illustration with his analogy to the nested Russian doll. Indeed, symmetry crops up everywhere including the insight that the act of “reducing” a thought to its simplest expression has the simultaneous effect of raising it to a higher level of generalization. This seems to be true across all levels of logical thought.

Slowly but steadily over the centuries the groundwork was laid, but it wasn't until as late as the 19th century that modern algebraic notation finally settled in, providing "relief for the imagination" so that algebra could really soar. The author takes several passes through this period so as to follow each branch of the subject and also points out the surprising and often unexpected ways these branches are connected.

Another interesting aspect of the story is that very few women have become true math pioneers; in fact only two make it into this book. Of whom one of these, Emmy Noether, a colleague quipped, "Emmy is certainly a great mathematician; but that she is a woman, I cannot swear." This naturally opens the question as to whether we can speak of a "female mind" just as there is a female body as distinct from the male body, or whether women are genetically predisposed to use their minds differently than men.

I know from my own, albeit limited, undergraduate math experience that in general men seemed more at ease with math than I was. But I also observed I could handle word problems much more readily than my male colleagues could. What this means I don't know, nor does the author offer any explanation, but we certainly cannot rule out the possibility of a sex-determined genetic predisposition for certain kinds of symbolic thought. Musical ability likewise seems to run in families and so must have something to do with genetics, but there seems no sense in complaining about *that*. The Larry Summers imbroglio shows us just how deep the denial of mundane reality really is among the educators and how utterly disconnected with the rest of us they have become, living lives apart and even hostile to the societal whole, where realities like male/female differences must be coped with.

Another basic question running through the book is: what *is* math exactly and how does it relate to mind? 150 years ago, according to the author in another piece, the Dutch philosopher [Jacob Moleschott said](#), "the brain secretes thought as the liver secretes bile," and today many people no doubt believe this kind of limited materialistic outlook, but is it the whole picture? Could we be at a point in our mental evolution like the ancients who struggled with admitting irrational numbers into their thoughts? Are we afraid of what we cannot see? And might we

not be living in multidimensional world constructed on the Russian doll model, with material reality held within the realm of mindal reality, so that we are forced to travel into the realm of pure mind (pure mathematics) in order to correctly describe the workings of material reality? This is the world mathematics describes. The question is, is this world so described, more than just a random flight of fancy collectively followed by a few esoteric math junkies?

Tom Wolfe [recently argued](#) that rather than the use of tools, it is the actually the use of symbolic language that defines man and separates him from beast. Animals do not have the ability to describe their inner state, nor are they truly self-conscious in the human sense with full consciousness of their consciousness. Indeed, the ability to describe oneself implies at least some degree of transcendence of self.

One might even argue that man is able to access levels of mind animals simply cannot reach and that it is these levels, these outer Russian dolls, which contain language potential for the human mind to activate. Language (including math) could be described as pre-installed software just waiting for each human being to plug in his or her own culturally inherited word-symbols. It is certainly doubtful that thought, in the human sense, is even possible at all without language.

Human languages all seem to work in certain logical ways just as does mathematics and it is interesting to note Mr. Derbyshire is a linguist as well as a mathematician. Both these disciplines concern, more or less directly, the study of the mind and how it functions. Richard Weaver describes the use of language as akin to riding a horse in the sense that one must learn what language will and won't do. Mathematics, then, is akin to soaring with the birds into realms where time and space are transcended and infinity is glimpsed. It even seems to circle back around and lend credence to ancient wisdom long since abandoned such as the Platonic concept of "forms." Quoting Mr. Derbyshire:

The two great 20th-century revolutions in physics were of course those that go under the heading of relativity and quantum theory. Both depended on concepts from 19th-century "pure" algebra.

Item. In the special theory of relativity, measurements of time and space made in one frame of reference can be “translated” to measurements made in another (traveling of course at constant velocity relative to the first) by means of a Lorentz transformation. These transformations can be modeled as rotations of the coordinate system in a certain four-dimensional space – in other words, as a Lie group.

Item. In general relativity this four dimensional time-space is distorted – curved – by the presence of matter and energy. For the proper description of it we must rely on *tensor calculus*, developed by the Italian algebraic geometers...

Item. When the young physicist Werner Heisenberg, in the spring of 1925, was working on the radiation frequencies emitted by an atom that “jumps” from one quantum to another, he found himself looking at large square arrays of numbers, the number in the n th column of the m th row in an array being the probability that the atom would jump from state m to state n . The logic of the situation required him to multiply these two arrays together and suggested the only proper technique for doing so, but when he tried to carry out this multiplication, he found that it was noncommutative. Multiplying array A by array B gave one result; multiplying B by A gave a different result. What on earth was going on? Fortunately, Heisenberg was a research assistant at the University of Gottingen, so he had David Hilbert and Emmy Noether on hand to gently explain the principles of matrix algebra.

Item. By the 1960s, physicists had uncovered a bewildering zoo of the type of nuclear particles called hadrons. Murray Gell-Mann, a young physicist at Caltech, noticed that the properties of hadrons, though they did not follow any obvious linear pattern, made sense in the context of another Lie group, one that appears when we study rotations in two-dimensional space whose coordinates are complex numbers. Working the data, Gell-Mann then saw that this original impression was superficial. The equivalent Lie group in a space of three complex dimensions had greater explanatory power. It required the existence of particles not yet observed, though. Gell-Mann published his results, experimenters powered up their particle colliders, and the predicted particles were duly observed. (pgs. 316-17)

And listen to this discussion of Calabi-Yau manifolds:

..These are six-dimensional spaces that, according to string theory, lurk in the tiniest regions of space-time, down to the Planck length (that is, a billionth of a trillionth of a trillionth of a centimeter)..The simplicity of their curvature – a certain kind of “smoothness” – makes them ideal for the kinds of string motions that, according to string theory appear to our instruments as all the varieties of subatomic particles and forces, including gravitation. The fact of their being six-dimensional is a bit alarming, but these “extra” dimensions are “folded up” out of sight from our vantage point up here in the macroscopic world, just as a thick three-dimensional hawser looks one-dimensional when viewed from sufficiently far away.

Here we are in [Jules Verne territory](#) where science meets imagination; where imagination is required to expand and enrich our understanding of reality. The gift Mr. Derbyshire gives in this book is the knowledge that it has always been so. We feel safe slipping the gravity grasp of space and time as we travel through dimensions of a purely mindal landscape because we are with someone who knows the terrain and we are deposited back on earth feeling exhilarated and free.

So don't be afraid, you *can* read Unknown Quantity. And not only that: it's *fun*. So go ahead, take the ride, you'll be better for it.