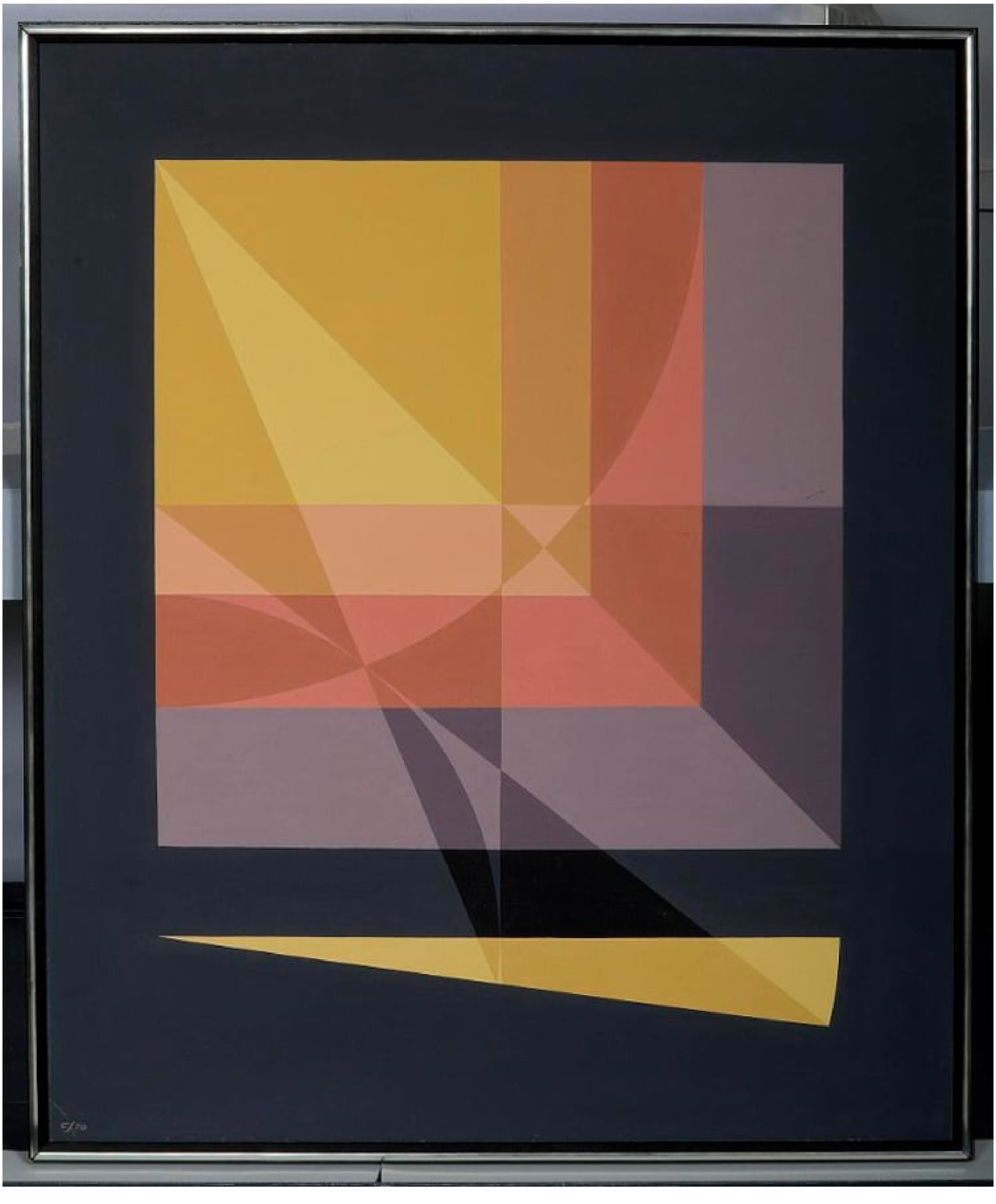
How did Mathematics Lose its Lustre?

by Christopher Ormell (November 2023)



Problem of Delos Constructed from a Solution by Isaac Newton (Arithmetica Universalis), Crockett Johnson, 1970

Most reflective people are aware that mathematics has lost its lustre, though few professional academic mathematicians, who have devoted their lives to finding new arcane theorems, will agree. Math has, in effect, gone thoroughly out of fashion. Like many former, once lauded, difficult activities, it retains its most dedicated followers, some of whom are hardly aware that their vocation has now sunk a long way down the pecking order. The most telling sign of this decline is probably that when the media were debating whether 'truth' meant anything—following the arrival of a new incumbent in the White House in 2017—nobody remarked that the most striking and certain truths were to be found in *math*. (They were things like the impossibility of squaring the circle, the impossibility of finding a fraction equal to the square-root of 2, or the fact that there are prime numbers larger than the googol or any other giant number you care to name.) The media and most ordinary people stopped taking much notice of 'math' and 'mathematics' some years ago. They have come to treat it as a relic of a by-gone age.

Why? How did this happen?

Well, the simple answer is of course that computing—which was originally a cuckoo in the nest of mathematics—took over, ousting the mathematicians from (a) the academic limelight, (b) the media and (c) the corridors of power. How could math, a venerable, ancient subject which had been around for more than 2,000 years, hope to compete with the pizazz and glamour of modern digital electronics?

But the truth is not quite as simple as this, if only because the digital computer was invented by two brilliant mathematicians (Alan Turing and John von Neumann), and because its intended function was to *automate* mathematics … a change which would only increase math's range and power.

So the truth is that mathematics has not become obsolete. It has, rather, taken on a new, unexpected, manifestation, as the core base of computer software. This new ‘manifestation,’ though, is just a part of the activity of the Silicon Valley corporations, and their decision has been to play down its role, and block its public visibility.

This tendency to talk-down mathematics began to bite in the 1980s when computer salespeople were trying to sell their first PCs to the general public. The salespeople went round assuring customers that “Computers have nothing to do with math!” They knew that there was a lot of mathsphobia out there, and that, among ordinary people, the very mention of ‘math’ touched a nerve. They were trying to create clear blue water between the new PCs and math.

They and their companies also knew that there was little enthusiasm for computers in academic math circles. (This attitude has slowly completely reversed over the sixty years which have passed since then.) Computers may have enlarged the range and power of math, but the academics weren’t—at that time—interested in the slightest in range and power, i.e., instrumental potency *vis-à-vis* the real world. Their view of math was that it explored a superior, lofty, precise, elegant, structured, timeless ‘Abstract Reality.’ Their attitude towards instrumental math was dismissive. They treated it as essentially a lowly, materialistic, utilitarian, tedious, substandard variant of their far superior ‘higher discipline.’

So the arrival of computers was not met with any kind of welcome from most of the math academics. In effect the main academic math community were intent on washing their hands of any involvement with instrumental math. They made it known that they were quite happy to let the computer sector take instrumental math over.

Was this a misjudgment? Yes, their dismissive condemnation of instrumental math was neither wise, nor realistic, nor well rooted. They seemed to be unaware that instrumental math generated a lot of priceless *illumination* of technical problems and physical theory. This was certainly not 'materialistic' in the usual sense, nor was it tedious, utilitarian, or substandard. The academics were mostly unaware too that the general public's previous high regard for math had been mainly grounded on historic examples of its illuminative effect, in science, technology, innovation and defence. They had no inkling that in schools the rejection of the instrumental side of math meant that the subject lost its applicative thunder—and hence much needed classroom credibility.

Today, here in the UK, academic math has largely morphed into the mathematical modelling of problems arising from science and technology. Some number theory—previously regarded as a form of 'pure maths' —is also being practised, the reason being that part of it (prime number theory) underlies most modern commercial and military cryptography.

But in spite of this de facto switch of research towards forms of advanced illuminative instrumental maths, there has been no conscious public 'moment of truth' creating visible distance between today's thinking and the subject's former semi-mystical line about 'Abstract Reality.' This metaphysical rhetoric should have been dismissed in the 1960s, but there is still no evident stomach to dump it. Why? Why has treating math as the 'study of Abstract Reality' been retained and believed so long after it lost the plot?

The answer to these questions is that there is a very deep semantic gulf between the math hierarchy and the general educated public. There is relatively little awareness in math circles of the decline of the subject's public standing, and virtually no awareness among the public of what happens in math circles, especially non-instrumental 'higher math' circles.

But behind these opaque communication walls there is a stark fact. It is that math encountered a string of rarely admitted, rarely mentioned internal crises in the 20^{th} century. (They are rarely mentioned because each time the math leadership's response was to hush the crisis up.) Mathematics, the subject which has been largely responsible for the state of the modern world, has been trying to grapple—and not very successfully—with various mind-boggling, existential setbacks since the 1900s. This is arguably the ultimate source of the contradictions which are subliminally sapping our morale and vitality today.

Even before 1900, the mathematic leadership was walking on thin ice. There was a fashionable movement in the late 19^{th} century intent on trying to put the whole of maths on a unified basis, a set theory basis. But sets generally, as such, could not be bona fide mathematical objects, because they often had physical objects as their elements. The collection of cows in farmer Giles's meadow undoubtedly form a set. But this "set" is not—it cannot be—a mathematical object. Mathematic objects can't be *milked*! A set could only be a bona fide mathematical object if all its elements were already mathematical objects, e.g. things like the googol (10^{100}), the squareroot of *-1*, the Mersenne numbers less than a million, or runs like 7777777 in the decimal expansion of p.

This distinction is absolutely essential if one is intent—as many logicians in the 1890s probably were—on using set theory to clarify mathematical thinking. Mathematics is a language whose objects stand out as well-defined symbolic conglomerations and well-defined symbolic processes. The least we can expect of its logicians, is that they recognise the crucial distinction between mathematical language and ordinary language. (This distinction does not, of course, prevent us from using formal math analogies to understand the real world better. The math mimics a real-world situation.)

But if the elements of a mathematical set must already be mathematical objects, it follows that a set cannot be the most basic concept in mathematics. One must know what a ‘mathematical object’ is, *before* one can even define a single mathematical set. It follows that the fashionable 19th century movement to put maths on a set basis was, and is, fundamentally flawed. I’m afraid that its illustrious exponents such as Frege, Russell, Whitehead, Zermelo … were trying to operate as mathematical logicians without first bothering to think-through how mathematical language relates to—is different from—ordinary language. As a result, they overlooked a howler among their own assumptions. This is a setback which was blithely overlooked at the time.

Another setback in academic mathematics occurred when the leaders of the academic subject tacitly accepted Cantor’s transfinite theory in 1900. (It postulated that there were cardinal super-infinities unimaginably greater than ordinary infinity.) The decision to canonise this notion created a rift in academic mathematics, because some mathematicians of the highest calibre, such as Kroneker (earlier), Poincare, Borel, and Weyl pointed out that the universe of discourse of all possible definable mathematic objects only contains an ordinary infinity of items. (This means that the immense (unimaginable) numbers of additional mathematic objects needed to populate Cantor’s postulated transfinite sets, could only be regarded as being indefinable, shadowy and, frankly, fairy-like. They were postulated objects, incidentally, which no mathematician would ever manage to see, write down or know. The fact that their indefinability implied ineligibility and unknowability was quietly forgotten.)

A third setback came about when the subject’s leadership—strongly influenced by the Cantorian theory they had so enthusiastically endorsed—decided at about the same time that higher maths was “should be recognised as” an “Intellectual Artform,” not a “science”. (The Aesthetic

Movement was dominant in philosophy around 1900, and the math gurus seemed to want to join this influential bandwagon.) They had correctly realised that mathematics cannot be an empirical science. The objects of mathematics are—from a sensible viewpoint at least—created as consensus symbolic conventions, much as laws, university degrees and treaties are initially created. They are no more part of the real world than the Kings and Queens of Chess are flesh-and-blood human beings.

Whichever way one interprets this, it was a bold departure from previous practice, and its hidden effect was to play down the primacy of rigour. The seriousness of this implication only became plain decades later, when Stanley Ulam realised that higher maths had diversified itself into something very like a standstill: it had vastly over-produced huge numbers (millions) of obscure, impenetrable theorems, the whole body of which no single person could ever possibly understand, even in an outline fashion. It presented a kind of terminally baffling, irremediable, overall incoherence.

A fourth severe setback was Russell’s Contradiction of 1901, which established that the set “of all sets which were not members of themselves” was necessarily both a member of itself and not a member of itself! But it couldn’t be both. This was an utterly unexpected, devastating development: math was destroying its own meaning. Left unexplained, it was a time bomb under the credibility of mathematics.

A fifth setback was Zermelo-Fraenkel set theory, which was widely adopted in the 1920s as a suggested way to defuse Russell’s Contradiction. It included an axiom which pronounced (ex-Cathedra) that a set could never be a member of itself. This was trying to talk Russell’s Contradiction out of existence. But there were sets, such as the set of all sets mentioned in this essay, which were *quite obviously* members of themselves. So the leading gurus of the subject knew perfectly well that their “fix” had provoked another contradiction, but they decided to brazen it out anyway … by *imposing* these axioms onto their colleagues as the official math ‘Party Line.’

A sixth setback was the arrival of the reliable digital computer around 1960. The huge boost it gave to the “instrumental” side of math roundly contradicted the assumption of the academic hierarchy, which had previously assumed that *the real significance of maths lies in its origin as the language God must have used when He created the universe* … (This was always a dubious presumption, because the timeless, rigid, infinitely static objects of math are pretty *unlike* most real objects—especially living organisms and human beings. Actually we can never know for sure whether there is a single ‘timeless object’ in the real world: because there is no way in which its verification could be completed. The knock-out punch however is that recently a much better alternative 100% abstract language, anti-mathematics, has been discovered.)

A seventh setback was ‘New Maths for Schools’ which was backed to the hilt by the US mathematic establishment in the early 1960s and which quickly spread to most advanced countries. It tried to switch the theme of maths education in schools from a focus on numbers, to a focus on sets. It then proceeded to collapse abysmally in schools all around the world, and was later judged to have seriously harmed the math education of a generation of young mathematicians: a conference of academic mathematicians in the UK in 1972 reached this conclusion. This seriously ill-advised ‘revolution’ had already thoroughly demoralised the wider world of education and much else besides.

An eighth setback was Ulam’s Dilemma, which, as we have already noted, Stanley Ulam arrived-at in 1976. He realised that there were several million higher maths research papers which had been published in reputable journals. The vast majority were sitting unread on the shelves of university libraries and Research Institutes. They were focused onto a huge diversity of novel, unexpected, higher maths notions. This meant that, apart from their authors and a few of their authors' colleagues, no one—however clever—could find the time to understand them. It was as if higher maths had stultified itself into incoherence by an uncontrolled explosion of way-out, idiosyncratic, and ivory-tower ideas. Higher maths had evidently lost the plot, i.e., any hope of having a coherent story to tell of "what it was all about".

So the total picture, when these eight setbacks are taken into account, would have been quite serious if they had been openly admitted. But because they were all assiduously covered-up, the damage done to math's reputation is more severe.

Has it led to a flight from non-instrumental math?

Yes, in effect, because most of the maths being researched in universities is now illuminative instrumental math. But the leading gurus of the academic subject have shown little appetite to re-define their subject or legitimise their U-turn.

They managed to convince themselves for centuries that *maths is the language of physical reality* when its rigidity and timelessness conspicuously contradicts the texture, vitality and destructive power of physical reality let alone the marvels of living creatures and reflective humanity. They thought that instrumental math was about fixing minor practical problems. They imagined that there was a free-standing "world" of Abstract Objects out there, quite independent of the human race.

This was a naïve, literal reading of the reifications used in math. It was guyed by Lewis Carroll when the Red Queen observed that Alice must have "wonderful eyesight" because she could see *nobody* coming down the road.

Many older mathematicians don't take any notice. They simply carry on doing what they like best—solving difficult abstract problems. They make no attempt to look at the big picture in a responsible, constructive way. They seem to think that if they don't look at this quagmire, it might go away. Thus the leadership of the academic subject seems to have fallen into a state of deeply suppressed denial and dread: one which can only foretell a problematic future. Consequently, they have not been able to find the confidence or the energy to repair the broken state of math in schools. A new start is obviously needed, one based on a proper understanding of what math contributes to the human race. It needs to come with a clear realisation that math can be pushed too hard, and that in relation to science anti-math is potentially a more credible, promising language for modelling a universe which contains immense explosive chaos as well as living things and reflective human beings.

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